Disturbance rejection of a non-contact 3-DOF stage using Gaussian process regression

The University of Tokyo Koseki-Ohnishi Lab
REON SASAKI
r.sasaki@ctl.t.u-tokyo.ac.jp

Non-contact 3-DOF stage

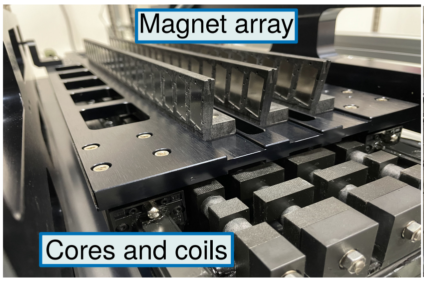
High-speed, high-precision positioning control

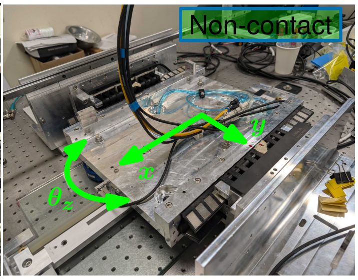
$\Rightarrow$ related to **production speed** and **quality**

$\Rightarrow$ Demand for actuators with **high thrust density**

$\Downarrow$

Motor with IRON CORES





State-dependent disturbance

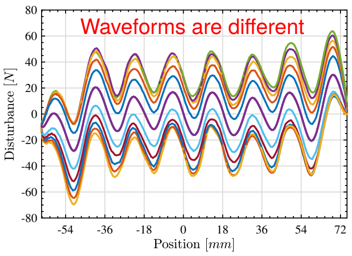
One of the major problems is... **Thrust Ripple!!**

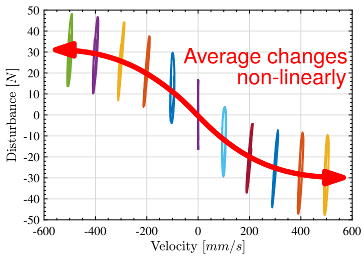
- **Cogging force** between iron cores and magnets
  depends on **Position** and **Current** $f_{\mathrm{d}}(x, u)$
- **Deviation of force constant** by relative position
  depends on **Position** and **Velocity** $f_{\mathrm{fc}}(x)$
- **Changes in viscosity friction** due to assembly errors
  depends on **Position** and **Velocity** $f_{\mathrm{vd}}(x, \dot{x})$

$$M\ddot{x} + D\dot{x} + \underbrace{f_{\mathrm{vd}}(x, \dot{x})}_{\text{Viscosity damping ripple}} + \underbrace{f_{\mathrm{d}}(x, u)}_{\text{Cogging ripple}} = (1 + \underbrace{f_{\mathrm{fc}}(x)}_{\text{Force constant variation}})u$$

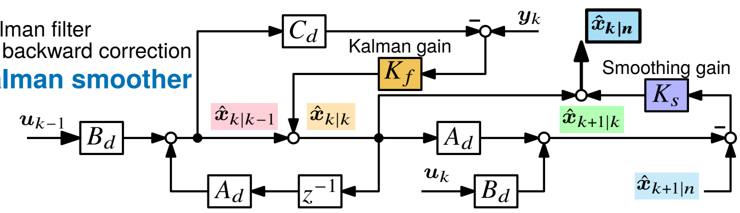
complicated...

State-depenent disturbance changes in various trajectories!





Kalman filter
+ backward correction
Kalman smoother



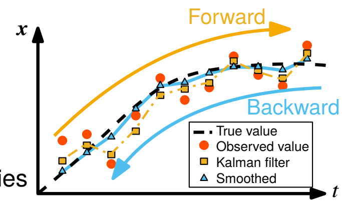
Prediction part (Kalman filter)

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_f(y_k - C_d\hat{x}_{k|k-1})$$

Smoothing part (RTS smoother)

$$\hat{x}_{k|n} = \hat{x}_{k|k} + K_s(\hat{x}_{k+1|n} - \hat{x}_{k+1|k})$$

Estimate disturbances in some trajectories



Reducing calculation cost of Gaussian process regression

Gaussian Process Regression (GPR)
Nonparametric bayesian estimation method
$\Rightarrow$ Prediction of outputs without basis functions.

Kernel function

$$k(\boldsymbol{x_n}, \boldsymbol{x_{n'}}) = \theta_1 \exp\left(-\frac{|p_n - p_{n'}|^2}{\theta_2} - \frac{|v_n - v_{n'}|^2}{\theta_3}\right) + \theta_4 \delta(n, n')$$

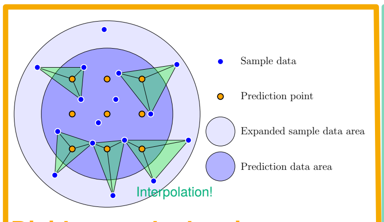
$\mu(\boldsymbol{x}^*) = K_{xx^*}^{\top}(K_{xx} + \sigma_n^2 I_N)^{-1}$ $\longrightarrow$ Predicting output of $\boldsymbol{x}^*$

$\Sigma(\boldsymbol{x}^*) = K_{x^*x^*} - K_{xx^*}^{\top}(K_{xx} + \sigma_n^2 I_N)^{-1}K_{xx^*}$ $\longrightarrow$ Reliability of prediction

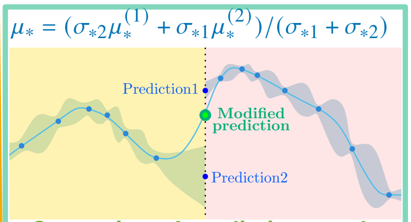
Size of matrix is decided by the **number of data**

Handling more data $\Rightarrow$ Increasing calculation time

To reduce calculation cost...

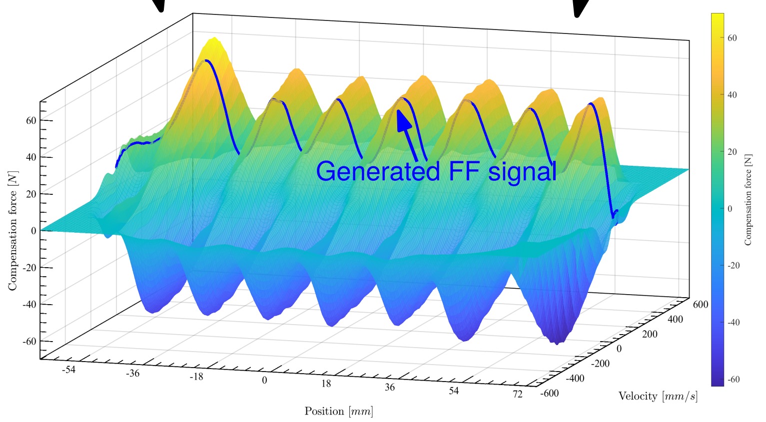


Divide sample data in segments



Correction of prediction result

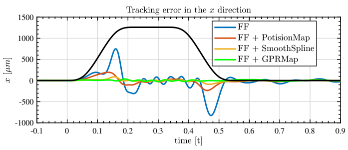
Make a **FORCE MAP**
for disturbance compensation

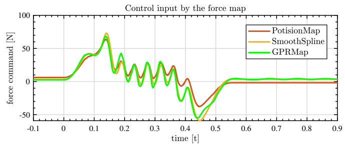


Disturbance compensation using the force map made by GPR

Compared the following error between some method

Model inversion vs Linear interpolation vs Smoothing spline vs GPR





	Max error	L2 norm
Case1	$749.5\mu m$	15710.2
Case2	$194.1\mu m$	5043.6
Case3	$88.3\mu m$	1734.2
Case4	$36.5\mu m$	593.1

Future work

- Use adopted disturbance observer
- Generate FF signal for all axis

Acknowledgement : **Koichi Sakata**, **Pai-Hsueh Yang** (Nikon)