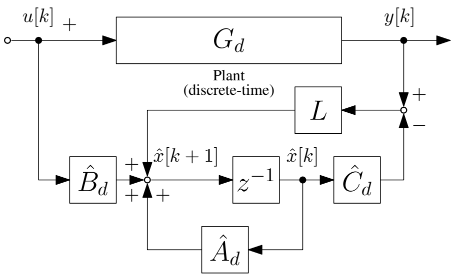
A non-causal approach for suppressing the estimation delay of state observer

Kentaro Tsurumoto*, Wataru Ohnishi*, Takafumi Koseki*, Nard Strijbosch**, Tom Oomen**,***
*The University of Tokyo **Eindhoven University of Technology *** Delft University of Technology

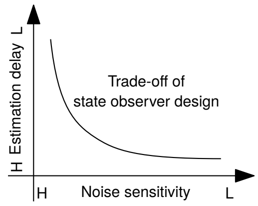
Background and conceptual idea

State observer



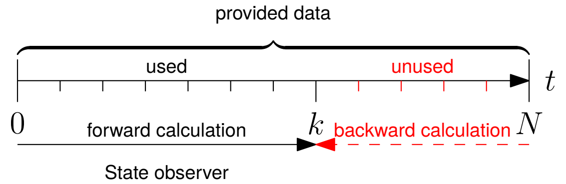
$$\hat{x}[k+1] = \hat{A}_d\hat{x}[k] + \hat{B}_d u[k] + L(y[k] - \hat{y}[k])$$
$$\hat{y}[k] = \hat{C}_d\hat{x}[k]$$

- ✓ sensorless
- ✓ estimation of unmeasurable variables
- ✗ estimation delay



Offline estimation of already executed systems

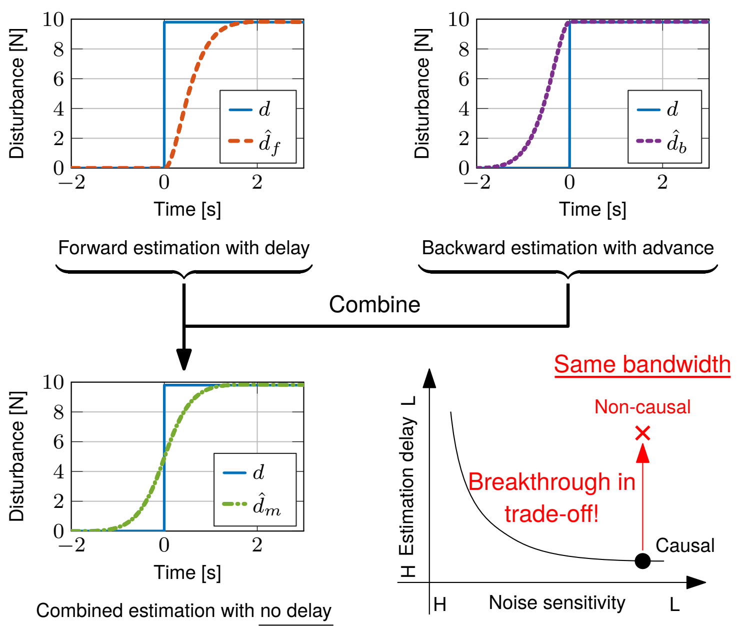
Simply applying the state observer does not utilize the whole data.
→ The already provided future data is not used!



Backwards estimation using future data has an estimation advance.

By combining an estimation with a delay and an estimaion with an advance, a suppressed delay estimation is realized!

Concept



Possible applications

- Offline phenomena analysis (e.g. cutting force estimation)
- Iterative Learning Control

Non-causal state observer

Backward estimation procedure

1. Construct an unstable state observer with a same bandwidth as the state observer used for the forward estimation.
2. Calculate the state equation backwards using stable inversion

$$\hat{x}[k] = \hat{A}_d^{-1}\left(\hat{x}[k+1] - \hat{B}_d u[k] - L(y[k] - \hat{y}[k])\right)$$
$$\hat{y}[k] = \hat{C}_d\hat{x}[k]$$

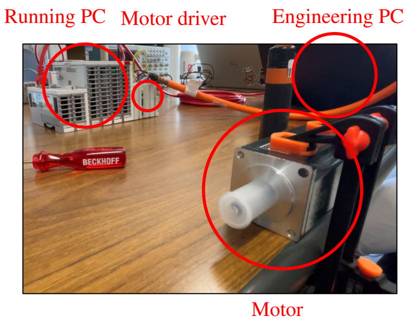
Composition of state estimation

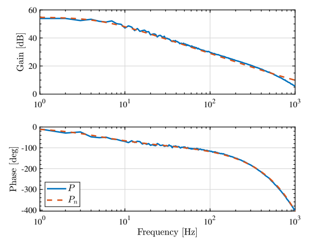
The forward estimation $\hat{x}_f$ and backward estimation $\hat{x}_b$ are combined based on their covariance matrix of estimation.
Assuming the covariance matrix being P_f and P_b, the combined non-causal estimation $\hat{x}_m$ is obtained by the following equation.

$$\hat{x}_m = \alpha\hat{x}_f + (I - \alpha)\hat{x}_b$$
$$\alpha = P_b(P_f + P_b)^{-1}$$

Experimental validation

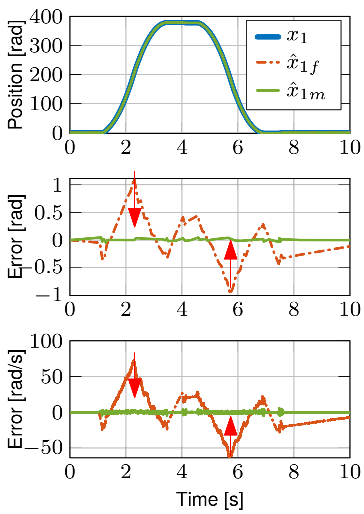
Experimental setup



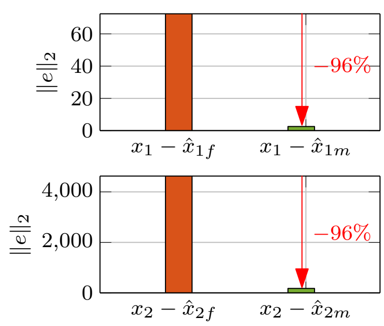


$$G_n = \frac{e^{-3T_s}}{5.65\times10^{-5}s^2 + 1.81\times10^{-3}s}$$
T_s = 0.25 ms

Controllable canonical form
$x_1 \cdots$ Position
$x_2 \cdots$ Velocity



State estimation result



Euclidean norm of error

Conclusion

Novelty → Application of stable inversion to an unstable observer, and combining it with an ordinary observer to achieve an effective state estimation for non-causal operations.

Future work → Application to Iterative Learning Control.

Contact: Tsurumoto Kentaro (tsuruken@koseki.t.u-tokyo.ac.jp)